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Transverse structure function in the factorisation method

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ABSTRACT

Deep Inelastic scattering experiments using transversely polarised targets yield information on the structure function g_2. By means of a free-field analysis, we study the operator structure of g_2 and demonstrate the need for retaining the twist three mass terms in order to maintain current-conservation. We show that the structure function $g_T = g_1 + g_2$ has a much simpler operator structure as compared to g_2, in spite of the fact that, like g_2, g_T has a twist-three component. We demonstrate factorisation of the hadronic tensor into hard and soft parts for the case of g_T. We show that the first moment of the gluonic contribution to g_T vanishes, and discuss possible physical applications.

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The recent transversely polarised deep inelastic scattering (DIS) experiments [1] have opened up new avenues to explore the polarised structure of the proton. With more data, better accuracy and new proposed experiments [2], it would soon be possible to extract the twist-three contribution for the first time. Though extraction of the twist-three contributions is in general quite difficult[3, 5], in these experiments it is possible to kinematically eliminate the leading twist contribution [6].

The polarised structure of the proton is characterised by two structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$ which can be measured in a polarised lepton-proton DIS experiment $\ell(k)P(p) \to \ell(k')X(p_x)$. The spin dependent part of the proton tensor is parametrised as

$$\widetilde{W}_{\mu\nu}(x,Q^2) = \frac{i}{p\cdot q}\epsilon_{\mu\nu\lambda\sigma}\, q^\lambda \left\{ s^\sigma \left(g_1(x,Q^2)+g_2(x,Q^2)\right) - \frac{q\cdot s}{p\cdot q}\, p^\sigma g_2(x,Q^2)\right\} \ , \quad (1)$$

where s_μ is the spin vector of the proton and is normalised as $s^2 = -M^2$ with $s\cdot p = 0$, M being the target mass. The spin-dependent cross-section [6] is given by

$$\begin{aligned} \frac{d\Delta\sigma(\alpha)}{dxdyd\phi} &= \frac{e^4}{4\pi^2 Q^2}\left\{ \cos\alpha \left\{ \left[1-\frac{y}{2}-\frac{y^2}{4}(\kappa-1)\right] g_1(x,Q^2) - \frac{y}{2}(\kappa-1) g_2(x,Q^2)\right\}\right. \\ &- \left. \sin\alpha\cos\phi\sqrt{(\kappa-1)\left(1-y-\frac{y^2}{4}(\kappa-1)\right)}\left[\frac{y}{2}\, g_1(x,Q^2)+g_2(x,Q^2)\right]\right\} \ , \quad (2) \end{aligned}$$

where $y = p\cdot q/p\cdot k$, $\kappa = 1+4x^2M^2/Q^2$, ϕ is the azimuthal angle and α is angle between the spin vector s and the incoming lepton momentum k. In a longitudinally polarised experiment ($\alpha = 0$), the dominant contribution comes from the structure function $g_1(x,Q^2)$ while $g_2(x,Q^2)$ is suppressed by a factor M^2/Q^2, thus enabling the extraction of $g_1(x,Q^2)$. The longitudinally polarised DIS process has been studied quite extensively [5, 7] and there is a considerable amount of data on $g_1(x,Q^2)$ [8]. In contrast, the extraction of $g_2(x,Q^2)$ requires transversely polarised proton ($\alpha = 90$) and further this cross-section is suppressed by a factor $M/\sqrt{Q^2}$ relative to the longitudinal case. Note that at the cross-section level the transverse asymmetry measures the twist-three contribution while the longitudinal asymmetry measures the twist-two contribution. Hence the extraction of $g_2(x,Q^2)$ is much more complicated as compared to $g_1(x,Q^2)$. Recently, experimental information on $g_2(x,Q^2)$ has become available [1], but the data have large errors and do not provide a definite answer to the question of the validity of the sum-rules associated with $g_2(x,Q^2)$, like the Burkhardt-

Cottingham (BC) sum-rule [9], the Wandzura-Wilczek (WW) sum-rule [10], or the recently proposed Efremov-Leader-Teryaev (ELT) sum-rule [11]. contribution etc.

In transversely polarised DIS experiments, the asymmetry that is measured is the virtual photon absorption asymmetry

$$A_2(x,Q^2) \quad = \quad \frac{\sqrt{Q^2}}{\nu}\ \frac{g_1(x,Q^2)+g_2(x,Q^2)}{F_1(x,Q^2)}\ , \tag{3}$$

where $F_1(x,Q^2)$ is the spin-averaged structure function. We see from eq. 3 that the asymmetry is proportional not to g_2 alone, but to $g_T(x,Q^2) = g_1(x,Q^2)+g_2(x,Q^2)$. In this letter, we show that the quantity g_T admits of a much simpler description than does g_2. We suggest that this may help in going some way towards a fuller understanding of the transverse spin structure of the nucleon. We begin by discussing the free field theory analysis [12] in order to elucidate the operator structure of the structure functions and demonstrate the importance of the mass term in maintaining gauge invariance of the hadronic tensor. We then discuss the first moment of $g_T(x,Q^2)$ and its relation to the spin content of the proton, and study the gluonic contribution to the first moment, using the Factorisation Method (FM) [13].

The hadronic tensor $W_{\mu\nu}(p,q,s)$ has the form

$$W_{\mu\nu}(p,q,s) = \frac{1}{4\pi}\int d^4\xi\ e^{iq.\xi}\ \langle ps|\ [J_\mu(\xi),\ J_\nu(0)]\ |ps\rangle_c\ , \tag{4}$$

Retaining the dominant contribution in the light-cone limit, $\xi^2 \to 0$, identified as the most singular part of the time-ordered product of these currents on the light-cone, we find

$$\widetilde{W}_{\mu\nu}(p,q,s) \quad = \quad \frac{i}{4\pi^2}\epsilon_{\mu\nu\lambda\rho}\int d^4\xi\ e^{iq.\xi}\ \xi^\lambda\ \delta^{(1)}(\xi^2)\ \epsilon(\xi_0)\ \langle ps|\ {:}\mathcal{O}^\rho_A(\xi,0){:}\ |ps\rangle_c\ , \tag{5}$$

where

$$\mathcal{O}^\rho_A(\xi,0) = \bar\psi(\xi)\gamma^\rho\gamma_5\psi(0) + \bar\psi(0)\gamma^\rho\gamma_5\psi(\xi)\ , \tag{6}$$

To arrive at the above result we used

$$iS(\xi,0) \quad = \quad -\langle 0|T(\psi(\xi)\bar\psi(0))|0\rangle\ , \tag{7}$$

$$\quad = \quad -\frac{i}{2\pi^2}\frac{\not\xi}{(\xi^2-i\epsilon)^2} + O(m)\ , \tag{8}$$

where order m terms are neglected. The importance of these terms will be shown later. To find the dominant contribution coming from these operators, we still have to make them

local and then pick up the dominant part. Hence,

$$\mathcal{O}^{\rho}_{A}(\xi,0)=\sum_{n}\frac{1}{n!}\xi^{\mu_1}\cdots\xi^{\mu_n}\mathcal{O}^{\rho}_{\mu_1\cdots\mu_n}(0)\ . \tag{9}$$

As we can see, the above local operator is symmetric in $\mu_1....\mu_n$ but has no definite symmetry in the permutation of ρ with any of the other indices. The dominant part of the above operator can be obtained, in the usual twist analysis, by decomposing into symmetric and mixed symmetric parts,

$$\mathcal{O}^{\rho}_{\mu_1\cdots\mu_n}=\mathcal{O}^{\{\rho}{}_{\mu_1\mu_2\cdots\mu_n\}}+\frac{2n}{n+1}\mathcal{O}^{\{[\rho}{}_{\mu_1]\mu_2\cdots\mu_n\}}\ . \tag{10}$$

The fully symmetric part is twist-two and the mixed symmetric part is twist-three. (As usual, $\{\ \ \}$, $[\ \ ]$ mean symmetrisation and antisymmetrisation respectively).

Let us now compute the twist-two contribution to $\widetilde{W}_{\mu\nu}(p,q,s)$. Expanding the operator matrix element in terms of the vectors available in the theory, the most general expression which is fully symmetric can be written as

$$\xi^{\mu_1}\cdots\xi^{\mu_n}\langle ps|\mathcal{O}^{\{\rho}{}_{\mu_1\mu_2\cdots\mu_n\}}|ps\rangle \;=\; \frac{B_n(p^2)}{(n+1)!}\left[n!\ s^{\rho}(\xi\cdot p)^n+n\ n!\ p^{\rho}\xi\cdot s(\xi\cdot p)^{n-1}\right]\ , \tag{11}$$

where $B_n(p^2)$ are unknown scalars which contain all the non-perturbative information.

To perform the integration in eqn.(5) using the delta function, we define the function $g(y)$ such that

$$b(y)=\frac{1}{2\pi}\sum_{n=0}\frac{B_n(p^2)}{(n+1)!}\int d(p\cdot\xi)\ e^{-iy\xi\cdot p}\ (\xi\cdot p)^n\ , \tag{12}$$

Substituting eqn.(11) in eqn.(5) and using the above Fourier decomposition we can perform the integrals and the result is

$$\widetilde{W}^{(2)}_{\mu\nu}=\frac{i}{4\ p\cdot q}\ \epsilon_{\mu\nu\lambda\rho}\ q^{\lambda}\left[s^{\rho}-p^{\rho}\frac{s\cdot q}{p\cdot q}\left(1+x\frac{d}{dx}\right)\right]b(x)\ . \tag{13}$$

Comparing the above expression with eqn.(1), we find

$$g_1^{(2)} \;=\; \frac{1}{4}\left(-x\frac{d}{dx}\right)b(x)\ , \tag{14}$$

$$g_2^{(2)} \;=\; \frac{1}{4}\left(1+x\frac{d}{dx}\right)b(x)\ , \tag{15}$$

where the superscript 2 denotes the twist of the operators contributing to the structure functions. Note that current conservation is maintained, i.e $q^\mu \widetilde{W}_{\mu\nu} = 0$. Let us now compute the twist-three contribution to $\widetilde{W}_{\mu\nu}(p,q,s)$. Again using symmetry arguments we find that

$$\begin{aligned} \xi_{\mu_1} \cdots \xi_{\mu_n} \mathcal{O}^{\{[\rho}_{\;\;\mu_1]\mu_2\cdots\mu_n\}} &= D_n(p^2)\ \xi_{\mu_1} \cdots \xi_{\mu_n}\ S^{\{[\rho} p^{\mu_1]} \cdots p^{\mu_n\}} \\ &= \frac{D_n(p^2)}{2}\ (s^\rho p^\alpha - s^\alpha p^\rho)\ \xi_\alpha (\xi \cdot p)^{n-1}\ . \end{aligned} \tag{16}$$

Substituting this in eqn.(5) and using

$$d(y) = \frac{1}{2\pi i} \sum_{n=0} \frac{n}{(n+1)!} D_n(p^2) \int d(p\cdot\xi)\ e^{-iy\xi\cdot p}\ (\xi \cdot p)^{n-1}\ , \tag{17}$$

we find that

$$\widetilde{W}^{(3)}_{\mu\nu} \;=\; \frac{i}{4\ p\cdot q}\ \epsilon_{\mu\nu\lambda\rho}\ \left[q^\lambda \left(\frac{s.q}{p.q} p^\rho - s^\rho \right) \frac{d}{dx} + p^\lambda s^\rho \left(1 - x\frac{d}{dx} \right) \right] d(x)\ , \tag{18}$$

where the superscript on the hadronic tensor denotes the twist. From the above equation it is clear that the second term does not satisfy the current conservation relation. It is important to realise that this non-conservation does not manifest only for the hadronic matrix elements, but continues to hold even if we were to compute the eqn.(5) between quark states. Doing this, we find

$$\widetilde{W}_{\mu\nu} = \frac{im}{2p.q} \epsilon_{\mu\nu\lambda\rho} s^\rho \Big((p+q)^\lambda \epsilon(q^0 + p^0)\delta(1-x) + (q-p)^\lambda \epsilon(q^0 - p^0)\delta(1+x) \Big)\ . \tag{19}$$

Notice that current conservation is violated even at this level. It can be maintained if we include the mass term which we dropped in the expansion of the time-ordered product. The mass term turns out to be

$$S^{(m)}(\xi) = \frac{1}{4i\pi^2} \frac{m}{\xi^2 - i\epsilon}\ . \tag{20}$$

Adding this term to the equation (8) and using the equation of motion for the quark fields, we get the manifestly current conserved form:

$$\widetilde{W}_{\mu\nu} = \frac{1}{8\pi^2} \epsilon_{\mu\nu\lambda\rho} q^\lambda \int d^4\xi e^{iq.\xi} \delta(\xi^2) \epsilon(\xi_0) \langle p,s| : \mathcal{O}^\rho(\xi,0) : |p,s\rangle\ . \tag{21}$$

We find that the mass term exactly cancels the current non-conserving part appearing in the eqn.(5) to reproduce the above current conserved equation. The above analysis shows us that when we work at a given order in the twist expansion, it is important to keep both

the singular terms and the regular terms that can contribute at the given order. The final result that we have when we combine the twist-2 and the twist-3 contributions is as follows:

$$g_1(x) = \frac{1}{4}\left[-x\frac{d}{dx}\, b(x) - \frac{p^2}{p.q}\left(1 + x\frac{d}{dx}\right) d(x)\right] , \tag{22}$$

$$g_2(x) = \frac{1}{4}\left[\left(1 + x\frac{d}{dx}\right) b(x) - \frac{d}{dx}\, d(x)\right] . \tag{23}$$

Here, one can easily see that $g_1(x, Q^2)$ and $g_2(x, Q^2)$ are related by WW sum-rule if the higher twist terms such as terms proportional to p^2 and $d(x)$ are neglected. ¿From the expression for $g_2(x, Q^2)$, it is interesting to note that the twist-two part $b(x)$ and the twist-three part $d(x)$ contribute to the cross-section at the same order in $M/\sqrt{Q^2}$. Since they appear at the same order, it is very difficult to disentangle these operators and, in general, a measurement of $g_2(x)$ will be sensitive to both twist-two and twist-three operators at the same level. While twist-two operators have a simple parton model interpretation, higher twist operators cannot be described with the same simple picture. As a consequence, $g_2(x, Q^2)$ is not amenable to a parton model interpretation. We shall see, in the following, that there is yet another problem with the interpretation of $g_2(x)$, *viz.*, the usual factorisation of the hadronic tensor into hard and soft parts, wherein there is a cancellation of the infrared and collinear singularities, does not occur for the case of g_2. On the contrary, if we consider g_T instead of g_2, then factorisation can, indeed, be demonstrated. In what follows, we proceed to verify these statements, using the factorisation method.

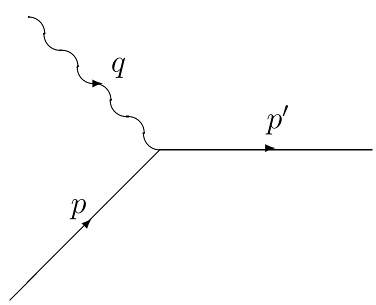

Fig. 1. Born diagram.

The factorisation theorem [13] ensures the separation of long distance (soft) effects from the short distance (hard) effects and hence in the DIS limit the quark and gluonic contribu-

tion to the polarised hadron tenor $\widetilde{W}^{\mu\nu}$ can be factorised as

$$\begin{aligned}\widetilde{W}^{\gamma^* P}_{\mu\nu}(x, Q^2) &= \sum_i \int_x^1 \frac{dy}{y} f_{\Delta q/P}(y, \mu^2)\ \widetilde{H}^{\gamma^* q_i}_{\mu\nu}(q, yp, \mu^2, \alpha_s(\mu^2)) \\ &\quad + \int_x^1 \frac{dy}{y} f_{\Delta g/P}(y, \mu^2)\ \widetilde{H}^{\gamma^* g}_{\mu\nu}(q, yp, \mu^2, \alpha_s(\mu^2))\ , \end{aligned} \tag{24}$$

where i runs over the quark flavours and μ is the factorisation scale which defines the separation of short distance from the long distance part. The soft effects are contained in the parton distribution functions $f_{\Delta a/P}$ which are proton matrix elements of certain gauge invariant bilocal operators made out of parton fields such as quarks and gluons. The hard scattering coefficients (HSC), $\widetilde{H}^{\gamma^* a}_{\mu\nu}$ are perturbative and the factorisation theorem in the DIS limit ensures that they are free of any infrared (IR) and collinear singularities and do not depend on the properties of the target. This target independence can be used to advantage: the HSCs can be computed order by order by replacing hadron states by asymptotic parton states. The contribution to various structure functions can be extracted by using appropriate projection operators.

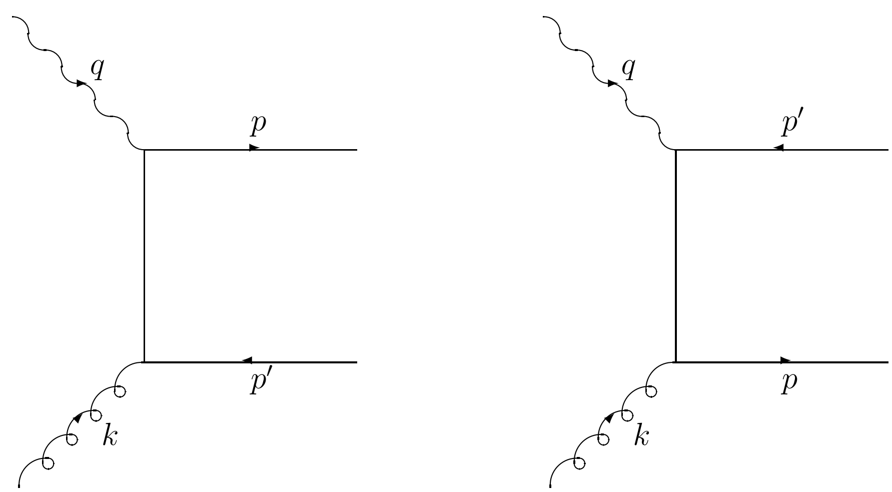

Fig. 2. Photon gluon fusion diagram

Let us begin by evaluating the HSCs to the quark sector to leading order. Replace the

proton by quark target ($P \to q$) and retain terms up to order $\mathcal{O}(\alpha_s^0)$ in eqn.(24)

$$\widetilde{W}_{\mu\nu}^{(0),\gamma^* q}(x,Q^2) \;=\; \sum_i \int_x^1 \frac{dy}{y} f^{(0)}_{\Delta q/q}(y,\mu^2)\ \widetilde{H}_{\mu\nu}^{(0),\gamma^* q_i}(q, yp, \mu^2, \alpha_s(\mu^2)) \;, \tag{25}$$

where the superscript in the above equation denotes the order of strong coupling α_s. $\widetilde{W}_{\mu\nu}^{(0),\gamma^* q}$ to $\mathcal{O}(\alpha_s^0)$ gets contribution from the Born diagram $\gamma^* q \to q$ (Fig. 1). For quarks as target, the distribution function can be calculated from the operator definitions (see below). To $\mathcal{O}(\alpha_s^0)$, $f^{(0)}_{\Delta q/q} \propto \delta(1-z)$ and hence $\widetilde{H}_{\mu\nu}^{(0),\gamma^* q}$ is same as the Born diagram. This is the usual statement that to leading order the parton model (PM) and the factorisation method which is a field theoretical generalisation of the PM are equivalent.

We now discuss factorisation at the next-to-leading order. To evaluate $\widetilde{H}_{\mu\nu}^{\gamma^* g}$ in eqn.(24), we replace $P \to g$, i.e

$$\begin{aligned}\widetilde{W}_{\mu\nu}^{(1),\gamma^* g}(x,Q^2) \;&=\; \sum_i \int_x^1 \frac{dy}{y} f^{(1)}_{\Delta q/g}(y,\mu^2)\ \widetilde{H}_{\mu\nu}^{(0),\gamma^* q_i}(q, yp, \mu^2, \alpha_s(\mu^2)) \\ &\quad + \int_x^1 \frac{dy}{y} f^{(0)}_{\Delta g/g}(y,\mu^2)\ \widetilde{H}_{\mu\nu}^{(1),\gamma^* g}(q, yp, \mu^2, \alpha_s(\mu^2)) \;,\end{aligned} \tag{26}$$

The LHS is the subprocess cross-section $\gamma^* g \to q\bar{q}$ (Fig. 2) and the RHS has two parts *viz.* the quark and gluon sector. We have shown that $\widetilde{H}_{\mu\nu}^{(0),\gamma^* q}$ is the subprocess $\gamma^* q \to q$. Hence we need to evaluate $f^{(1)}_{\Delta q/g}$ and $f^{(0)}_{\Delta g/g}$ from the definitions given in eqn.(27,28), by replacing $P \to g$. To $\mathcal{O}(\alpha_s^0)$, $f^{(0)}_{\Delta g/g} \propto \delta(1-z)$. To evaluate $\widetilde{H}_{\mu\nu}^{(1),\gamma^* g}$, we have to evaluate $\widetilde{W}_{\mu\nu}^{(1),\gamma^* q}$ and $f^{(1)}_{\Delta q/g}$. The above procedure works fine for the unpolarised structure functions $F_{1,2}(x,Q^2)$ [13] and the longitudinally polarised structure function $g_1(x,Q^2)$ [15, 17], but the transversely polarised structure function $g_2(x,Q^2)$ turns out to be an exception. Projecting the contribution to $g_2(x,Q^2)$ in eqn.(25), it turns out that $H_2^{(0),\gamma^* q} = 0$. This is expected as we know from the PM that $g_2(x,Q^2) = 0$ to leading order. As a consequence it turns out from eqn.(26) that $\widetilde{W}_2^{(1),\gamma^* g} = \widetilde{H}_2^{(1),\gamma^* g}$. Hence the usual cancellation of IR and collinear singularities between the subprocess cross-section and the appropriate parton matrix element, to give a HSC free of these singularities does not *seem* to occur in the case of $g_2(x,Q^2)$. The reason for this is that the above expression is incomplete as far as the extraction of $g_2(x,Q^2)$ is concerned. In fact the operator structure for the $g_2(x,Q^2)$ is much more complicated than that of $g_1(x,Q^2)$. Also, the simple minded convolution may not work for $g_2(x,Q^2)$. This has earlier been demonstrated using free field analysis.

Let us now demonstrate the claim of factorisation made for the case of g_T. To *leading twist*, $g_T(x, Q^2)$ gets contribution from the parton distributions functions, *viz.*

$$
\begin{aligned}
f_{\Delta q/P}(x,\mu^2) &= \frac{1}{4\pi}\int d\xi^- e^{-ix\xi^- p^+}\Big[\langle ps_\perp|\bar{\psi}_a(\xi^-)\ \not{s}_\perp\gamma_5\mathcal{G}^a_b\psi^b(0)|ps_\perp\rangle_c \\
&\quad +\langle ps_\perp|\bar{\psi}_a(0)\ \not{s}_\perp\gamma_5\mathcal{G}^a_b\psi^b(\xi^-)|ps_\perp\rangle_c\Big]\ , \qquad (27)\\
f_{\Delta g/P}(x,\mu^2) &= \frac{i}{4\pi x p^+}\epsilon_{\mu\nu\lambda\sigma}\ s^\lambda p^\sigma\int d\xi^- e^{-ix\xi^- p^+}\Big[\langle ps_\perp|F^{+\mu}(\xi^-)\mathcal{G}^a_b F^{+\nu}(0)|ps_\perp\rangle_c \\
&\quad -\langle ps_\perp|F^{+\mu}(0)\mathcal{G}^a_b F^{+\nu}(\xi^-)|ps_\perp\rangle_c\Big]\ , \qquad (28)
\end{aligned}
$$

where the light-cone variables have been used to denote any four vector $\xi^\mu = (\xi^+, \xi^-, \xi_T)$, with $\xi^\pm = (\xi^0 \pm \xi^3)/\sqrt{2}$. $F^{\mu\nu}_a$ is the gluon field strength tensor and $\mathcal{G}^a_b \equiv \mathcal{P}\exp[ig\int_0^{\xi^-} d\zeta^- A^+(\zeta^-)]^a_b$ is the path ordered exponent which restores the gauge invariance of the bilocal operators. We do not consider the other twist-three gluonic operator [18] that contributes to DIS as they are suppressed by strong coupling. For parton targets the above distributions are normalised as

$$
\begin{aligned}
f_{(\Delta q+\Delta\bar{q})/a(h)}(z) &= h\ \delta(1-z)\ \delta_{a,(q,\bar{q})}\ , \qquad (29)\\
f_{\Delta g/a(h)}(z) &= h\ \delta(1-z)\ \delta_{a,g}\ , \qquad (30)
\end{aligned}
$$

where $h = \pm 1$ is the helicity of the incoming parton and z is the sub-process Björken variable.

Using the procedure discussed above, we evaluate the HSCs to transversely polarised structure function $g_T(x, Q^2)$ and study its factorisation properties. To $\mathcal{O}(\alpha_s^0)$ we can project the $g_T(x, Q^2)$ contribution from eqn.(25). The LHS is the Born diagram $\gamma^*(q)q(p) \to q(p')$ (Fig. 1) and its contribution to $g_T(x, Q^2) \neq 0$. Using the normalisation condition eqn.(29) we find

$$
\widetilde{H}_T^{(0),\gamma^* q} = \frac{e^2}{2}\ \delta(1-z)\ , \qquad (31)
$$

From eqn.(24) it is clear that to leading order $g_T(x, Q^2)$ gets contribution from the parton distribution eqn.(27). At next to leading order $g_T(x, Q^2)$ gets contribution from the other parton distribution eqn.(28). To evaluate the corresponding HSC $\widetilde{H}_T^{(1),\gamma^* g}(x, Q^2)$, we use eqn.(26) and project out the $g_T(x, Q^2)$ contribution. This involves the calculation of the matrix element eqn.(27) between gluon states $f^{(1)}_{\Delta q/g}$ and the sub process cross-section $\widetilde{W}_T^{(1),\gamma^* g}$, both to $\mathcal{O}(\alpha_s)$.

The sub-process cross-section $\widetilde{W}_{\mu\nu}^{(1),\gamma^* g}$ involves the $\gamma^*(q)g(k) \to q(p)\bar{q}(p')$ fusion process (Fig 2). This diagram is free of UV divergence but has a mass singularity, which appears at small scattering angles in the massless limit. We could regulate this by keeping either the quark or the gluon mass non-zero, but we choose to keep both particles massive as the prescription dependence would be explicit in this case. Projecting the contribution to $g_T(x,Q^2)$, by using the appropriate projection operator, we get

$$\begin{aligned}\widetilde{W}_T^{(1),\gamma^* g} &= e^2 \frac{\alpha_s}{4\pi} \int_{-1}^{1} \frac{dL}{(1-L^2\omega^2\kappa)^2} \left\{ \left(-1 + z + \frac{k^2}{q^2} z \right) (1-L^2\omega^2)^2 - 4\frac{m^2}{q^2} z (1+L^2\omega^2) \right. \\ &\left. + 4\frac{k^2}{q^2} z^2 L^2 \omega^2 \left[-2(1-z) + 4\frac{m^2+k^2}{q^2} z - (1-z)(1-L^2\omega^2) - \frac{k^2}{q^2} z (1+L^2\omega^2) \right] \right\},\end{aligned}$$

where $\omega^2 = 1 - 4m^2/s$, $s = (q+k)^2$ and $L = \cos\theta$, θ being the centre-of-mass scattering angle. Performing the two body phase-space integral in the centre-of-mass frame, we get

$$\widetilde{W}_T^{(1),\gamma^* g} = e^2 \ \frac{\alpha_s}{2\pi} \ \frac{k^2 z (1-z)^2}{m^2 - k^2 z (1-z)} \ , \tag{32}$$

Note that the contribution to $g_T(x,Q^2)$ is independent of the $\ln Q^2$ term. Both $g_1(x,Q^2)$ and $g_2(x,Q^2)$ separately depend on the $\ln Q^2$, but the combination $g_T(x,Q^2)$ is independent of the $\ln Q^2$ term. The constant piece depends on the choice of regulator and hence is prescription dependent as is clearly seen.

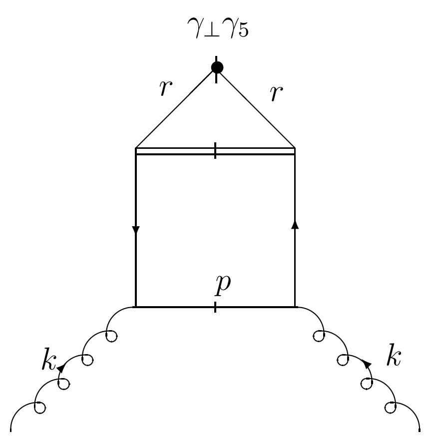

Fig. 3. The $\mathcal{O}(\alpha_s)$ contribution to the matrix $f_{\Delta q/g}$.

The matrix element $f_{\Delta q/g}^{(1)}$ (Fig. 3) is evaluated using the parton distribution eqn.(27) in the light-cone gauge with the replacement $P \to g$. We keep the quarks and gluon off

mass-shell as we did for the evaluation of cross-section. Noting that this matrix element is superficially divergent, we compute it using dimensional regularisation method, and the matrix element turns out to be

$$f^{(1)}_{(\Delta q+\Delta\bar{q})/g} = 2\alpha_s \int \frac{d^{d-2}p_\perp}{(2\pi)^{d-2}} \frac{1-z}{(p_\perp^2+m^2-k^2z(1-z))^2}\left[m^2+k^2z(1-z)+\frac{d-4}{d-2}p_\perp^2\right] . \quad (33)$$

As is clear, the integral is convergent and reduces to a simple form:

$$f^{(1)}_{(\Delta q+\Delta\bar{q})/g} = \frac{\alpha_s}{\pi} \frac{k^2z(1-z)^2}{m^2-k^2z(1-z)} . \quad (34)$$

The term $(d-4)/(d-2)$ in the integral gives non-vanishing finite contribution. We also checked the correctness of our result in the Pauli-Villars (PV) regularisation scheme. We could reproduce the same result in this scheme also confirming that our finite result is UV scheme independent. In the PV regularisation, since the integral is performed in four dimension, the $(d-4)/(d-2)$ term is absent. The analogous term comes from the integral with m replaced by M (PV regulator) in the limit M goes to infinity. Hence, our result is independent of UV scheme. This is not true in the case of operator matrix elements which one encounters in the evaluation of the structure functions $F_2(x,Q^2)$ and $g_1(x,Q^2)$ [15]. Recall that the matrix elements appearing in the evaluation of the QCD corrections to $F_2(x,Q^2)$ and $g_1(x,Q^2)$ are UV renormalisation scheme dependent. In other words, those operators are defined/renormalised in a definite UV renormalisation scheme say $\overline{MS}$ or momentum subtraction scheme or Pauli-Villars scheme. In our case, since the matrix element is finite the result is UV renormalisation scheme independent to this order. Observe that the masses we introduced to avoid IR singularities lead to two different results when one considers the cross-section and the matrix element separately. That is, both the cross-section and the matrix element are dependent on the order in which the masses go to zero. This is the usual prescription dependence one encounters in massless theories. The prescription-dependent structure of the above equation is the same as that of $W^{(1),\gamma^* g}$. Substituting for the normalisation condition eqn.(30) and eqn.(31,32,34) in eqn.(26), we get

$$\widetilde{H}_T^{(1),\gamma^* g} = 0 . \quad (35)$$

Note that there is a cancellation of the prescription dependent pieces confirming the factorisation. Also, it turns out that the next to leading order HSC, $\widetilde{H}_T^{(1),\gamma^* g}$ is zero and hence

twist-three distribution eqn.(28) does not contribute to $g_T(x,Q^2)$ to any of the moment to this order. The above analysis proves that the first moment of the gluon coefficient function is zero in FM.

One of the important outcomes of the demonstration of factorisation for g_T is that it admits a description in terms of a process-independent universal distribution. This distribution is no longer a parton distribution in the usual sense of the term, because of the twist-three contribution to g_T. But the process-independence is still useful, so that once the non-perturbative distribution associated with g_T has been extracted in one experiment (in DIS, for example), it can be used to make predictions for other processes, like Drell-Yan.

The above analysis also has interesting consequences for the first moment of the structure function $g_T(x,Q^2)$. At the leading order, one would be led by the validity of the BC sum-rule to conclude that the first moment of $g_T(x,Q^2)$ is same as that of $g_1(x,Q^2)$. Though these first moments are measured in completely different experiments ($g_T(x,Q^2)$ in transversely polarised DIS and $g_1(x,Q^2)$ in longitudinally polarised DIS), they should coincide numerically. Note that because the BC sum-rule is valid at one loop order [14], one would expect that the first moment of the gluonic coefficient vanishes in the FM. Our analysis of g_T using the factorisation method confirms this expectation. The first moments of both $g_1(x,Q^2)$ and $g_T(x,Q^2)$ in FM, are related to one and the same matrix element $\langle ps|\bar{\psi}\ \not{s}\gamma_5\psi|ps\rangle$ which is Lorentz invariant. The argument used here is the same as the rotational invariance argument that may be used to justify the BC sum-rule [4, 5].

A related issue is that of the hard gluonic contribution to the first moment of g_1 *via* the anomaly [19]. This gluonic contribution induced through the anomaly is, in fact, a possible explanation for the surprisingly small value for the first moment of g_1 measured in experiments [8]. In the FM [15], however, this contribution vanishes as long as the quark distributions are related to matrix elements of the standard quark field-operators which appear in the operator product expansion [16]. In a parton model computation of the hard gluonic contribution due to the anomaly, the gluonic coefficient is found to be non-zero [19]. In other words, the size of the gluonic contribution is dependent on the definition of the parton distributions. In the case where there is a non-vanishing hard gluonic contribution to the first moment of g_1 induced by the anomaly, our analysis would tell us that precisely the same contribution will also affect g_T. Thus, a measurement of the first moment of $g_T(x)$ will

provide a very interesting cross-check about the importance of the anomaly-induced gluonic contribution.

Another interesting prediction for g_T could be the analogue of the Björken sum rule for g_1. Given that the first moments of g_1 and g_T are identical, we would expect that g_T would satisfy a sum-rule which is exactly the same as the Björken sum-rule, and whose numerical value is the same as that for g_1.

In conclusion, we have shown that the transverse structure function $g_T(x, Q^2)(g_1(x, Q^2) + g_2(x, Q^2))$ contains a simple operator structure which renders one to understand the spin structure of the proton from a completely different experiment involving transversely polarised proton. In addition, due to the simplicity in its structure, it is easier to extract and hence understand the higher twist effects. We have shown that the first moment of $g_T(x, Q^2)$ measures the spin contributions coming from various partons to the proton spin using the Factorisation method. The interesting point to observe is that at large Q^2 to order $\alpha_s(Q^2)$, with appropriate operator definitions for transverse partons inside the transversely polarised proton, the factorisation of mass singularities works. We have found that the gluonic contribution to $g_T(x, Q^2)$ is zero to this order for the operators discussed.

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